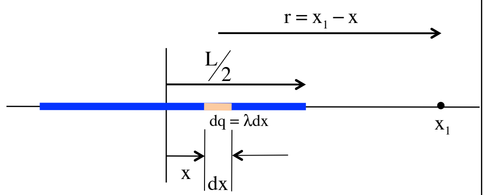
Problem 23.29



The rod has charge density λ and length L. We want the electric field at x_1. To that end, we need to determine the differential electric field dE generated at x_1 due to a differential bit of charge "dq" located at "x" on the rod. We then need to sum up via integration all the differential fields due to all the differential bits of charge in the rod. All the information needed to do the calculation is shown on the sketch. That calculation is shown on the next page:

$$E_x = \int dE = \int k\frac{dq}{r^2}$$

$$= k\int_{x=-L/2}^{L/2}\frac{\lambda dx}{(x_1 - x)^2}$$

$$= k\lambda\left(-\left(-\frac{1}{(x_1 - x)}\right)\right)\Bigg|_{x=-L/2}^{L/2}$$

$$= k\lambda\left(\frac{1}{(x_1 - L/2)} - \frac{1}{(x_1 - (-L/2))}\right)$$

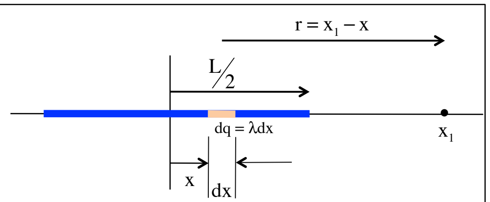
$$= k\lambda\left(\frac{1}{(x_1 - L/2)} - \frac{1}{(x_1 + L/2)}\right)$$

$$= k\lambda\left(\frac{x_1 + L/2}{(x_1 - L/2)(x_1 + L/2)} - \frac{x_1 - L/2}{(x_1 - L/2)(x_1 + L/2)}\right)$$

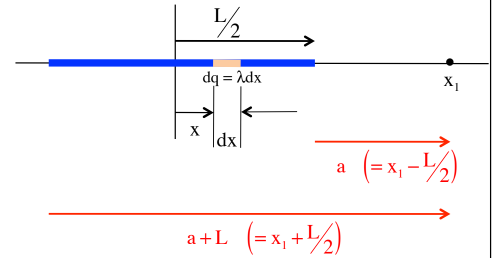
$$= k\left(\frac{Q}{\cancel{L}}\right)\left(\frac{\cancel{L}}{(x_1 - L/2)(x_1 + L/2)}\right)$$

$$= \frac{(9\text{x}10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)(22.0\text{x}10^{-6}\ \text{C})}{\left((.360\ \text{m}) - \left(\frac{.014\ \text{m}}{2}\right)\right)\left((.360\ \text{m}) + \left(\frac{.014\ \text{m}}{2}\right)\right)}$$

$$= (1.53\text{x}10^6\ \text{N/C})(-\hat{i})$$ (with the charge negative, the field is back toward the source)



Note: Notice that if we define the distance "a" as shown in the sketch, then "a+L" comes out as also shown. With those definitions, we can re-write our derived electric field function as:



$$E_x = \frac{kQ}{(x_1 - L/2)(x_1 + L/2)}$$

$$= \frac{kQ}{a(a+L)}$$

This is exactly the final form for the electric field function generated by a charged rod along its axis as derived by the book in Example 23.6.